

**Note**

# Tables of Divergent Feynman Integrals in the Axial and Light-Cone Gauges

## 1. INTRODUCTION

The evaluation and use of Feynman integrals in axial gauges is generic to many problems ranging from perturbation calculations of quantum field theories [1] to quantum gravity [2]. The chief advantage in choosing the axial gauge, defined by  $A \cdot n = 0$  with  $A$  being the gauge field and  $n$  an external vector, is that Faddeev-Popov ghosts [3] are not required to uphold Ward identities [4] in non-Abelian theories, so that the resulting calculations are simplified. The greatest disadvantage of this choice of gauge has been the technical difficulties encountered in the evaluation of the Feynman integrals that arise from such theories. In fact with the principal-value prescription [5] and dimensional regularization [6] which until recently have been the main means of evaluating such integrals, only a few tabulations of their infinite parts are extant [2, 7] in the literature, and fewer still representations of the divergent and regular parts of such integrals are known [8].

With the recent realization [9] that analytic regularization [10] is a viable technique for evaluating such integrals, the outlook has changed dramatically. It has been shown [9] that analytic regularization yields the same results as does the principal-value prescription, and that gauge invariance is preserved [11]. Additionally, analytic regularization yields a mechanistic method of evaluating all Feynman integrals of a certain type, and in fact it is possible to classify all such integrals in general [12].

With this classification, the evaluation of any such integral is well defined, and much easier to carry out than when the principal-value prescription is used, but is still tedious. The purpose of this paper is to present a tabulation of those integrals which commonly appear in contemporary field theories. In addition, we emphasize that with analytic regularization the tensor algebra endemic to such theories is defined once and for all, in contrast to dimensional regularization which requires a separate prescription for each class of tensor. Thus the following tabulation only lists integrals with scalar integrands. The evaluation of integrals containing tensor integrands can be performed by referring to the tabulations and invoking standard techniques of tensor algebra demonstrated elsewhere [11].

## 2. ANALYTIC REGULARIZATION

The class of massless, 2-point integrals under consideration is defined by

$$S_{2\omega}(p, n; \kappa, \mu, \nu, s) \equiv \int d^{2\omega}q [(p-q)^2]^\kappa (q^2)^\mu (q \cdot n)^{2\nu+s} \quad (2.1)$$

where  $\omega$ ,  $\kappa$ ,  $\mu$ , and  $\nu$  are arbitrary and continuous variables,  $s=0$  or  $1$ ,  $p$  is an external momentum and the gauge vector  $n$  was defined previously. We work with a Euclidean metric; defining the variable

$$y = (p \cdot n)^2/p^2 n^2$$

allows Minkowski space to be reached by analytic continuation to  $|y|>1$ . Whenever  $\kappa$ ,  $\mu$ , and  $\nu$  take on integer values  $K$ ,  $M$ , and  $N$ , respectively, and  $\omega=2$ , the integral  $S_4(p, n; K, M, N, s)$  is said to be a “primal” integral which may have arisen in some field theory and in general is divergent. The method of analytic regularization is based on the observation [12] that (2.1) has the analytic representation

$$\begin{aligned} S_{2\omega} &= \frac{\pi^\omega (p^2)^{\alpha_1} (n^2)^{\alpha_2} (p \cdot n)^s \Gamma(\alpha_2 + s + 1/2)}{\Gamma(\beta_1 - \alpha_0) \Gamma(\beta_1 - \alpha_1) \Gamma(-\alpha_0 - \alpha_1 - s) \Gamma(-\alpha_2)} \\ &\times G_{3,3}^{2,3}(y|_{0,\beta_1,1/2-s}^{1+\alpha_0,1+\alpha_1,1+\alpha_2}), \end{aligned} \quad (2.2)$$

where  $G$  is Meijer’s  $G$ -function [13] and

$$\begin{aligned} \alpha_0 &= -\mu - \nu - s - \omega, \\ \alpha_1 &= \kappa + \mu + \nu + \omega, \\ \alpha_2 &= \nu, \\ \beta_1 &= \kappa + \nu + \omega, \end{aligned} \quad (2.3)$$

composed of integer parts

$$\begin{aligned} A_0 &= -M - N - s - 2 \\ A_1 &= K + M + N - 2 \\ A_2 &= N \\ B_1 &= K + N + 2 \end{aligned} \quad (2.4)$$

and epsilons

$$\begin{aligned} \varepsilon_0 &= -\alpha_0 + A_0 = \sigma + \tau + \varepsilon \\ \varepsilon_1 &= -\alpha_1 + A_1 = -\rho - \sigma - \tau - \varepsilon \\ \varepsilon_2 &= -\alpha_2 + A_2 = -\tau \\ \varepsilon_3 &= \beta_1 - \alpha_2 + A_2 - B_1 = \rho + \varepsilon \end{aligned} \quad (2.5)$$

with infinitesimals  $\rho$ ,  $\sigma$ ,  $\tau$ , and  $\varepsilon$  defined via

$$\begin{aligned}\kappa &= K + \rho \\ \mu &= M + \sigma \\ v &= N + \tau \\ \omega &= 2 + \varepsilon.\end{aligned}\tag{2.6}$$

Although the general axial gauge is of interest, the use of the light-cone gauge defined by  $n^2 = 0^+$  simplifies computations. In this gauge, it has been shown [11] that a representation that preserves the gauge invariance of the theory can be deduced from (2.2). Specifically, we have

$$\begin{aligned}L_{2\omega}(p; \kappa, \mu, \bar{v}) &\equiv S_{2\omega}(p, n; \kappa, \mu, v, s)|_{n^2=0} \\ &= \frac{\pi^\omega(p^2)^{\omega+\kappa+\mu}(p \cdot n)^\bar{v} \Gamma(\omega+\kappa) \Gamma(\omega+\mu+\bar{v}) \Gamma(-\omega-\kappa-\mu)}{\Gamma(-\kappa) \Gamma(-\mu) \Gamma(2\omega+\kappa+\mu+\bar{v})},\end{aligned}\tag{2.7}$$

where  $\bar{v} = 2v + s$ .

To obtain an analytic representation for any primal integral, in both gauges, it is sufficient to let  $\tau = \rho = \sigma = 0$  and evaluate (2.2) or (2.7) in the limit  $\varepsilon \rightarrow 0$ . Divergences of the primal integral will appear as singularities (poles) in  $\varepsilon$ , and the regular part can be found from the nonleading terms ( $O(\varepsilon^0)$ ) of the expansion of (2.2) or (2.7) about  $\varepsilon = 0$ . In practice, we note that if  $n^2 \neq 0$ , poles in  $\varepsilon_1$  label ultraviolet divergences of the primal integral whereas poles in  $\varepsilon_0$  or  $\varepsilon_3$  label infrared divergences. In both gauges there are no singularities in  $\varepsilon_2$ .

In some applications [14] it is desirable to distinguish between the singularities. This is achieved in the tables by evaluating (2.2) in the limit  $\tau = 0$ ,  $\sigma = \rho$ , so that  $\varepsilon_0 = \varepsilon_3 \neq -\varepsilon_1$ , and the poles appear as singularities of the form  $\varepsilon_0^{-1}$  or  $\varepsilon_1^{-1}$ . In the evaluation of the regular parts, we set  $\rho = \sigma = \tau = 0$  so that here all epsilon dependence cancels.

### 3. ALGORITHM

The representation (2.2) can be conveniently written as a contour integral

$$\begin{aligned}S &= \frac{\pi^\omega(p^2)^{\omega_1}(n^2)^v \Gamma(s+v+1/2)(p \cdot n)^s}{\Gamma(-\mu) \Gamma(-v) \Gamma(-\kappa) \Gamma(2v+\mu+\kappa+s+2\omega)} \\ &\quad \times \frac{1}{2\pi i} \times \int_L dt y^t \\ &\quad \times \frac{\Gamma(-t) \Gamma(\omega+v+\kappa-t) \Gamma(\mu+v+s+\omega+t)}{\Gamma(-\mu-v-\kappa-\omega+t) \Gamma(-v+t)} \\ &\quad \times \frac{}{\Gamma(1/2+s+t)}\end{aligned}\tag{3.1}$$

where the negatively directed contour  $L$  encloses the poles of the first two gamma functions in the numerator and excludes the others. The generation of singularities from the location of poles, zeros and pinches of the contour is discussed elsewhere [12]. It is sufficient to note here that (3.1) can be evaluated by straightforward application of the residue theorem at those finite number of points where exterior and interior singularities of the integrand coalesce when  $\tau=0$  and  $\rho=\sigma$  with  $\varepsilon$  small. This has been done using the algebraic manipulator code SCHOONSCHIP [15].

Once the sum of residues of the integrand has been found in (3.1), it is possible to isolate the singular part of the integral. Rewrite all gamma functions in the residue of (3.1) or in (2.7) having the integer part of their argument negative as

$$\Gamma(-L+\varepsilon_i) = \frac{(-)^L \Gamma(1-\varepsilon_i) \Gamma(1+\varepsilon_i)}{\Gamma(1+L-\varepsilon_i) \varepsilon_i} \quad (3.2)$$

whenever  $L$  is a positive integer and  $i \in (0, 3)$ . After replacing all such gamma functions and setting  $\varepsilon_3 = \varepsilon_0$ , the resulting expression may contain poles in  $\varepsilon_0$  and/or  $\varepsilon_1$ . The regular part may be found by evaluating each of the terms, using for example

$$\begin{aligned} y^{L+\varepsilon_i} &= y^L (1 + \varepsilon_i \ln y + \dots), \\ \Gamma(L \pm \varepsilon_i) &= \Gamma(L) \pm \varepsilon_i \psi(L) + \dots, \end{aligned} \quad (3.3)$$

and letting  $\varepsilon_1 = -\varepsilon_0 = -\varepsilon$  whenever any term appears of order  $\varepsilon^0$ . It should be noted that the evaluation of “exponent derivatives” such as  $(\partial/\partial\kappa) S_{2\omega}(p, n; \kappa, \mu, \nu, s)$  can be found by using higher order forms of (3.3) and a prescription given elsewhere [12, 16]. The SCHOONSCHIP program SINTD used to generate the tables has this capability, but this aspect will not be discussed further here. In one instance, the sum of residues generates an infinite series which is labelled specially by the symbol  $Z$ . We find

$$\begin{aligned} Z &= 2 \sum_{l=0}^{\infty} \frac{(1)_l y^l}{(3/2)_l} [\ln y - \psi(1+l) + \psi(3/2+l)], & |y| \leq 1 \\ &= \frac{1}{2\sqrt{\pi}} \sum_{l=0}^{\infty} \frac{(1/2)_l y^{-l-1}}{(1)_l} \{(\psi(1/2-l) - \psi(1+l) - \ln y)^2 \\ &\quad + 2\psi'(1/2) - \psi'(1+l) - \psi'(1/2-l)\}, & |y| > 1 \\ &= \sqrt{\pi} G_{3,3}^{2,3}(y | \begin{smallmatrix} 0,0,0 \\ 0,0,-1/2 \end{smallmatrix}), & \forall y. \end{aligned} \quad (3.4)$$

## 4. RESULTS

Tables I–V list the infinite and regular parts of (2.2) and (2.7) for  $-2 \leq K \leq 4$ , and all values of  $M$  and  $N$  in the range  $-3 \leq M, N \leq 4$  together with  $s=0$  and  $s=1$ . Each case is labelled with square brackets  $[K, M, N, s]$  and subscript  $l$ , which, as a double check, satisfies

$$l = 27 + 64s + 8N + M.$$

In the general axial gauge (Tables I–III) we define

$$\hat{S}_l = \pi^{-\omega}(p^2)^{-K-M-2}(p \cdot n)^{-2N-s} S_4(p, n; K, M, N, s) \quad (4.1)$$

and

$$1/\hat{\epsilon}_0 = 1/\varepsilon_0 + \gamma + \ln p^2 \quad (4.2a)$$

$$1/\hat{\epsilon}_1 = 1/\varepsilon_1 + \gamma + \ln p^2. \quad (4.2b)$$

In the light cone gauge ( $n^2 = 0$ ), Tables IV and V, we use ( $\bar{N} = 2N + s$ )

$$\hat{L}_l = \pi^{-\omega}(p^2)^{-K-M-2}(p \cdot n)^{-\bar{N}} L_4(p; K, M, \bar{N}) \quad (4.3)$$

and

$$1/\hat{\epsilon} = 1/\varepsilon + \gamma + \ln p^2. \quad (4.4)$$

In (4.4), as opposed to (4.2), all poles are amalgamated into poles in  $\varepsilon$  because in the light-cone gauge infrared and ultraviolet divergences must not be distinguished [11, 14].

To guard against transcription errors, all output from SINTD was reformatted using an on-line editor to generate the tables which were typed directly by an on-line device, and photographically reproduced. Any integrals omitted in the range of variables considered are identically zero. Note that some integrals with  $K \geq 0$  and/or  $(M, N) \geq 0$  do not vanish when infrared divergences are distinguished from ultraviolet ones in the axial gauge. Those with  $K=0$  are the so-called “tadpole” integrals which are usually set to zero in the principal-value prescription. In the general axial gauge, several cases generated by SINTD were checked by comparison with the general results given in Table 1 of [12].

*Note added in proof.* For the light-cone gauge the integrals given here, which are based on the principal-value prescription, do not give a renormalizable theory [11]. Recently a representation similar to (2.2) for light-cone gauge integrals based on Mandelstam's prescription [*Nucl. Phys. B* 213 (1983), 149] has been found [CRNL, preprint, TP-84-X-24] and the renormalizability of the Yang-Mills theory in this prescription has been demonstrated [CRNL, preprint, TP-85-II-11]. A table of integrals based on the new prescription will be published at a later date.

TABLE I  
Axial Gauge Integrals with  $K = -2$

$\hat{s}_{0, -13/30}$	$\hat{s}_{0, -13/30} = 137/30 - 21y/5 + 168y^2/5 + 8496y^3 - 191616y^4 + 420134y^5/5 - 18694144y^6/15 + 2955264y^7/5 + 1/\hat{e}_0 + \log(4y) * (1 + 1120y^3 - 40320y^4 + 225792y^5 - 401408y^6 + 221184y^7) + 1/\hat{e}_0 * (1 + 1120y^3 + 40320y^4 - 225792y^5 + 401408y^6 - 221184y^7)$	$\hat{s}_{1, -11/3} = -11/3 + \log(4y) + 1/\hat{e}_0$
$\hat{s}_{1, -13/30}$	$\hat{s}_{1, -13/30} = 137/30 - 12y/5 + 12y^2 + 8832y^3/5 - 23808y^4 + 290304y^5/5 - 552448y^6/15 + \log(4y) * (1 + 256y^3 - 5/60y^4 + 18432y^5 - 14336y^6) + 1/\hat{e}_0 * (1 - 256y^3 + 5/60y^4 + 14336y^5) + 1/\hat{e}_0 * (1 - 1120y^3 + 40320y^4 - 225792y^5 + 401408y^6 - 221184y^7)$	$\hat{s}_{2, -2, -2, 0} = -11/3 - 8y^2/3 + 108(4y) * (1 - 8y^2 + 32y^3) + 1/\hat{e}_0 * 1/\hat{e}_1 * (-8y^2 + 32y^3) + 1/\hat{e}_0 + 1/\hat{e}_1 * (1 + 8y^2 - 32y^3/3 + 208y^3/3)$
$\hat{s}_{-2, -1, -3, 0}$	$\hat{s}_{-2, -1, -3, 0} = -137/30 - y + 2912y^3/15 - 6528y^4/15 + 19073y^5/15 + \log(4y) * (1 + 223y^3 - 384y^4 + 512y^5) + 1/\hat{e}_0 * (1 - 32y^3 + 384y^4 - 512y^5) + 1/\hat{e}_0 * (137/30 + \log(4y) + 1/\hat{e}_0) - 137/30 + 3y/5 + \log(4y) + 1/\hat{e}_0 - 137/30 + 4y/5 + 4y^2/15 + \log(4y) + 1/\hat{e}_0 - 137/30 + 3y/5 - 163y^3/15 + \log(4y) + 1/\hat{e}_0 - 137/30 + 128y^3/15 - 192y^4/15 + \log(4y) + 1/\hat{e}_0 + 1/\hat{e}_0 * (1 + 160y^2 - 3200y^3 + 9600y^4 + 7168y^5) + 1/\hat{e}_0 * (1 + 48y^2 - 512y^3 + 4864y^4/3 - 4288y^4/3 + 108(4y) * (1 - 48y^2 + 512y^3 - 640y^4) + 1/\hat{e}_0 * (1 + 48y^2 - 512y^3 + 4864y^4/3 - 4288y^4/3 + 108(4y) * (1 - 8y^2 + 32y^3) + 1/\hat{e}_0 * (1 + 8y^2 - 32y^3) + 1/\hat{e}_0 + 1/\hat{e}_1 * (1 + 8y - 24y^2))$	$\hat{s}_{1, 1} = -11/3 + 2y/3 + \log(4y) + 1/\hat{e}_0$
$\hat{s}_{-2, -1, -2, 0}$	$\hat{s}_{-2, -1, -2, 0} = -11/3 - 16y/3 - 280y^2 + 4864y^3/3 - 4288y^4/3 + 108(4y) * (1 - 8y^2 + 32y^3) + 1/\hat{e}_0 * (1 + 8y^2 - 32y^3) + 1/\hat{e}_0 + 1/\hat{e}_1 * (1 + 8y - 24y^2)$	$\hat{s}_{1, 0} = -11/3 - 2y/3 + \log(4y) + 1/\hat{e}_0$

$[-2, 3, -1, 0]$	$\hat{s}_{22} = -2 + 12y + 120y^2 - 176y^3 + \log(4y) * (1 + 18y - 144y^2 + 160y^3) + 1/\hat{e}_0 + 1/\hat{e}_1 * (18y - 144y^2 + 160y^3)$	$[-2, -3, 2, 0]$	$\hat{s}_{40} = -3 + 3/(16y^2) + 3/(4y) + 1/\hat{e}_0$
$[-2, 4, -1, 0]$	$\hat{s}_{23} = -2 + 16y + 528y^2 - 5504y^3/3 + 4160y^4/3 + \log(4y) * (1 + 32y - 480y^2 + 1280y^3 - 896y^4) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-32y - 480y^2 + 1280y^3 - 896y^4)$	$[-2, -2, 2, 0]$	$\hat{s}_{41} = -17/6 + 5/(24y^2) + 1/y + 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(8y^2))$
$[-2, -3, 0, 0]$	$\hat{s}_{24} = -5 + 2/\hat{e}_0$	$[-2, 1, 2, 0]$	$\hat{s}_{42} = -25/12 - 1/(6y^2) + 7/(4y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (1/(16y^2) - 3/(4y))$
$[-2, -2, 0, 0]$	$\hat{s}_{25} = -2 + 2/\hat{e}_0$	$[-2, 2, 2, 0]$	$\hat{s}_{43} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, -1, 0, 0]$	$\hat{s}_{26} = 1/\hat{e}_0$	$[-2, 3, 2, 0]$	$\hat{s}_{44} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, 0, 0, 0]$	$\hat{s}_{27} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 4, 2, 0]$	$\hat{s}_{45} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, 1, 0, 0]$	$\hat{s}_{28} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, -3, 3, 0]$	$\hat{s}_{46} = -43/12 + 25/(192y^3) + 15/(32y^2) + 15/(16y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-5/(64y^3))$
$[-2, 2, 0, 0]$	$\hat{s}_{29} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, -2, 3, 0]$	$\hat{s}_{47} = -197/60 - 77/(480y^2) + 39/(40y^2) + 3/(2y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (1/(16y^3) - 9/(16y^2))$
$[-2, 3, 0, 0]$	$\hat{s}_{30} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, -1, 3, 0]$	$\hat{s}_{48} = -49/20 + 23/(480y^3) - 169/(160y^2) + 157/(48y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(64y^3) + 3/(8y^2) - 5/(4y))$
$[-2, 4, 0, 0]$	$\hat{s}_{31} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 0, 3, 0]$	$\hat{s}_{49} = -197/60 - 77/(480y^2) + 39/(40y^2) + 3/(2y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(16y^3) - 9/(16y^2))$
$[-2, -3, 1, 0]$	$\hat{s}_{32} = -3/2 - 1/(6y) + 1/\hat{e}_0 * (1 + 1/(4y))$	$[-2, 1, 3, 0]$	$\hat{s}_{50} = -49/20 + 23/(480y^3) - 169/(160y^2) + 157/(48y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(64y^3) + 3/(8y^2) - 5/(4y))$
$[-2, -2, 1, 0]$	$\hat{s}_{33} = -2 + 1/(2y) + 1/\hat{e}_0$	$[-2, 2, 3, 0]$	$\hat{s}_{51} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, -1, 1, 0]$	$\hat{s}_{34} = -3/2 + 1/(2y) + 1/\hat{e}_0 * (-1/(4y))$	$[-2, 1, 3, 0]$	$\hat{s}_{52} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, 3, 1, 0]$	$\hat{s}_{35} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 3, 3, 0]$	$\hat{s}_{53} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, 4, 1, 0]$	$\hat{s}_{36} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 4, 3, 0]$	$\hat{s}_{54} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, 2, 1, 0]$	$\hat{s}_{37} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, -3, 4, 0]$	$\hat{s}_{55} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[-2, 3, 1, 0]$	$\hat{s}_{38} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, -2, 4, 0]$	$\hat{s}_{56} = -79/20 - 539/(3840y^4) + 329/(480y^3) + 7/(8y^2) + 7/(6y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (7/(128y^4) - 7/(16y^3))$
$[-2, 4, 1, 0]$	$\hat{s}_{39} = 1/\hat{e}_0 - 1/\hat{e}_1$		$\hat{s}_{57} = -503/140 + 317/(4480y^4) - 109/(840y^3) + 799/(36y^2) + 2/y + 1/\hat{e}_0 + 1/\hat{e}_1 * (-3/(128y^4) + 1/(2y^3) - 5/(4y^2))$

TABLE I—Continued

$[-2, -1, 4, 9]$	$\hat{S}_{58} = -761/280 - 11/(840y^4) + 331/(672y^3) - 627/(224y^2) + 199/(40y)$ + $1/\hat{e}_0 + 1/\hat{e}_1 * (1/(258y^4) - 5/(32y^3) + 15/(16y^2) - 7/(4y))$	$[-2, -2, -2, 1]$	$\hat{S}_{73} = -3 - 12y + 240y^2 - 256y^3$ + $108(4y) * (1 + 72y^2 - 128y^3)$ + $1/\hat{e}_0 * (1 - 72y^2 + 128y^3)$ .
$[-2, 0, 4, 0]$	$\hat{S}_{59} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, -1, -2, 1]$	$\hat{S}_{74} = -3 - 4y + 16y^2 + \log(4y) * (1 + 8y^2)$ + $1/\hat{e}_0 * (1 - 8y^2)$
$[-2, 1, 4, 0]$	$\hat{S}_{60} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 0, -2, 1]$	$\hat{S}_{75} = -3 + \log(4y) + 1/\hat{e}_0$
$[-2, 2, 4, 0]$	$\hat{S}_{61} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 1, -2, 1]$	$\hat{S}_{76} = -3 + \log(4y) + 1/\hat{e}_0$
$[-2, 3, 4, 0]$	$\hat{S}_{62} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 2, -2, 1]$	$\hat{S}_{77} = -3 - 4y + 16y^2 + \log(4y) * (1 + 8y^2)$ + $1/\hat{e}_0 + 1/\hat{e}_1 * (8y^2)$
$[-2, 4, 4, 0]$	$\hat{S}_{63} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[-2, 3, -2, 1]$	$\hat{S}_{78} = -3 - 12y + 48y^2$ + $1/\hat{e}_0 * (1 + 72y^2 - 128y^3)$
$[-2, -3, -3, 1]$	$\hat{S}_{64} = -25/6 - 6y + 120y^2 - 16960y^3 + 1095440y^4 - 195840y^5 + 311296y^6/3$ + $\log(4y) * (1 - 3200y^3 + 28800y^4 + 64512y^5 + 40960y^6)$ + $1/\hat{e}_0 * (1 + 3200y^3 - 28800y^4 + 64512y^5 - 40960y^6)$	$[-2, 4, -2, 1]$	$\hat{S}_{79} = -3 - 24y + 48y^2 + 768y^3 - 960y^4$ + $1/\hat{e}_0 + 1/\hat{e}_1 * (72y^2 - 128y^3)$
$[-2, -2, -3, 1]$	$\hat{S}_{65} = -25/6 - 10y/3 + 40y^2 - 8768y^3/3 + 30080y^4/3 - 7424y^5$ + $\log(4y) * (1 - 640y^3 + 3200y^4 - 3072y^5)$ + $1/\hat{e}_0 * (1 + 640y^3 - 3200y^4 + 3072y^5)$	$[-2, -3, -1, 1]$	$\hat{S}_{80} = -32y + 24y^2$ + $1/\hat{e}_0 * (1 + 12y + 16y^2)$
$[-2, -1, -3, 1]$	$\hat{S}_{66} = -25/6 - 4y/3 + 8y^2 - 704y^3/3 + 896y^4/3$ + $\log(4y) * (1 - 64y^3 + 128y^4)$ + $1/\hat{e}_0 * (1 + 64y^3 - 128y^4)$	$[-2, -2, -1, 1]$	$\hat{S}_{81} = -4y + \log(4y) * (1 - 4y)$ + $1/\hat{e}_0 * (1 + 12y - 16y^2)$
$[-2, 0, -3, 1]$	$\hat{S}_{67} = -25/6 + \log(4y) + 1/\hat{e}_0$	$[-2, -1, -1, 1]$	$\hat{S}_{82} = \log(4y) + 1/\hat{e}_0$
$[-2, 1, -3, 1]$	$\hat{S}_{68} = -25/6 + 2y/3 + \log(4y) + 1/\hat{e}_0$	$[-2, 0, -1, 1]$	$\hat{S}_{83} = \log(4y) + 1/\hat{e}_0$
$[-2, 2, -3, 1]$	$\hat{S}_{69} = -25/6 + 2y/3 + 108(4y) + 1/\hat{e}_0$	$[-2, 1, -1, 1]$	$\hat{S}_{84} = 4y + \log(4y) * (1 - 4y)$ + $1/\hat{e}_0 * (1 + 4y)$
$[-2, 3, -3, 1]$	$\hat{S}_{70} = -25/6 - 32y^3/3 + \log(4y) + 1/\hat{e}_0$	$[-2, 2, -1, 1]$	$\hat{S}_{85} = 16y - 24y^2$ + $\log(4y) * (1 - 12y + 16y^2)$ + $1/\hat{e}_0 + 1/\hat{e}_1 * (-12y + 16y^2)$
$[-2, 4, -3, 1]$	$\hat{S}_{71} = -25/6 - 4y/3 + 8y^2 - 608y^3/3 + 896y^4/3$ + $\log(4y) * (1 - 64y^3 + 128y^4)$ + $1/\hat{e}_0 * (-64y^3 + 128y^4)$		
$[-2, -3, -2, 1]$	$\hat{S}_{72} = -3 - 24y + 1248y^2 - 3712y^3 + 2496y^4$ + $\log(4y) * (1 + 288y^2 + 1280y^3 + 1152y^4)$ + $1/\hat{e}_0 * (1 - 288y^2 + 1280y^3 - 1152y^4)$		

$\hat{s}_{46} = 36y - 424y^2/3 + 352y^3/3$	$\hat{s}_{107} = 1/\hat{e}_0 - 1/\hat{e}_1$
$+ \log(4y) * (-1 - 24y + 80y^2 - 64y^3)$	$-2, 0, 2, 1]$
$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-24y + 80y^2 - 64y^3)$	$\hat{s}_{108} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{47} = 64y - 464y^2 + 2752y^3/3 - 1600y^4/3$	$-2, 2, 2, 1]$
$+ \log(4y) * (-1 - 40y + 240y^2 - 448y^3 + 256y^4)$	$\hat{s}_{109} = 1/\hat{e}_0 - 1/\hat{e}_1$
$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-40y + 240y^2 - 448y^3 + 256y^4)$	$[-2, 3, 2, 1]$
$\hat{s}_{48} = -3 + 2/\hat{e}_0$	$\hat{s}_{110} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{49} = -1 + 1/\hat{e}_0$	$\hat{s}_{111} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{50} = -1 + 1/\hat{e}_0$	$\hat{s}_{112} = -227/60 + 329/(960y^3) + 21/(32y^2) + 21/(20y)$
$\hat{s}_{51} = 1/\hat{e}_0 - 1/\hat{e}_1$	$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-7/(32y^3))$
$\hat{s}_{52} = 1/\hat{e}_0 - 1/\hat{e}_1$	$\hat{s}_{113} = -69/20 - 539/(960y^3) + 763/(480y^2) + 7/(4y)$
$\hat{s}_{53} = 1/\hat{e}_0 - 1/\hat{e}_1$	$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-7/(32y^3))$
$\hat{s}_{54} = 1/\hat{e}_0 - 1/\hat{e}_1$	$\hat{s}_{114} = -363/140 + 1303/((6720y^3) - 1219(672y^2) + 1149/(280y))$
$\hat{s}_{55} = 1/\hat{e}_0 - 1/\hat{e}_1$	$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(16y^3) + 5/(8y^2) - 3/(2y))$
$\hat{s}_{56} = -9/2 + 3/(4y) + 1/\hat{e}_0$	$\hat{s}_{115} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{57} = -5/2 + 3/(4y) + 1/\hat{e}_0$	$\hat{s}_{116} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{58} = -11/6 + 13/(12y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(2y))$	$\hat{s}_{117} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{59} = 1/\hat{e}_0 - 1/\hat{e}_1$	$\hat{s}_{118} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{60} = 1/\hat{e}_0 - 1/\hat{e}_1$	$\hat{s}_{119} = 1/\hat{e}_0 - 1/\hat{e}_1$
$\hat{s}_{61} = 1/\hat{e}_0 - 1/\hat{e}_1$	$\hat{s}_{120} = -573/140 - 4761/((8960y^4) + 673/(560y^3) + 9/(8y^2) + 9/(7y))$
$\hat{s}_{62} = -3/2 + 3/(4y) + 1/\hat{e}_0$	$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (27/(128y^4) - 3/(4y^3))$
$\hat{s}_{63} = -11/6 + 13/(12y) + 1/\hat{e}_0 + 1/\hat{e}_1 * (-1/(2y))$	$\hat{s}_{121} = -1041/280 + 2853/((8960y^4) - 557/((224y^3) + 3753/((1120y^2) + 9/(4y)))$
$\hat{s}_{64} = 1/\hat{e}_0 - 1/\hat{e}_1$	$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-27/(256y^4) + 15/(16y^3) - 27/(16y^2))$
$\hat{s}_{65} = 1/\hat{e}_0 - 1/\hat{e}_1$	$\hat{s}_{122} = -7129/2520 - 1063/((16128y^4) + 2021/(2016y^3) - 1931/(480y^2) + 7409/(1280y))$
$\hat{s}_{66} = 1/\hat{e}_0 - 1/\hat{e}_1$	$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (5/(256y^4) - 5/(16y^3) + 21/(16y^2) - 2/y)$
$\hat{s}_{67} = -10/3 + 5/(16y^2) + 5/(6y) + 1/\hat{e}_0$	$[-2, 0, 4, 1]$
$\hat{s}_{68} = -37/12 + 25/(48y^2) + 5/(4y)$	$\hat{s}_{123} = 1/\hat{e}_0 - 1/\hat{e}_1$
$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-5/(16y^2))$	$[-2, 1, 4, 1]$
$\hat{s}_{69} = -13/60 - 41/(80y^2) + 149/(60y)$	$[-2, 2, 4, 1]$
$+ 1/\hat{e}_0 + 1/\hat{e}_1 * (-3/(16y^2) - 1/y)$	$[-2, 3, 4, 1]$
	$[-2, 4, 4, 1]$
	$\hat{s}_{125} = 1/\hat{e}_0 - 1/\hat{e}_1$
	$\hat{s}_{126} = 1/\hat{e}_0 - 1/\hat{e}_1$
	$\hat{s}_{127} = 1/\hat{e}_0 - 1/\hat{e}_1$

TABLE II  
Axial Gauge Integrals with  $K = -1$

$\hat{S}_0 = -y/5 + 12y^2/5 + 3632y^3/5 - 17792y^4 + 81792y^5$	$\hat{S}_{13} = -2y/3 - 8y^2 + 16y^3$
$- 624128y^6/5 + 301036y^7/5$	$+ \log(4y) * (-8y^2/3 + 32y^3/3)$
$+ \log(4y) * (-96y^3 - 3840y^4 + 23040y^5 - 43200y^6 + 24576y^7)$	$+ 1/\hat{e}_1 * (-8y^2/3 + 32y^3/3)$
$+ 1/\hat{e}_0 * (-96y^3 + 3840y^4 + 23040y^5 + 43200y^6 - 24576y^7)$	$\hat{S}_{14} = -2y/3 - 16y^2 + 16y^3 + 64y^4/3$
$\hat{S}_1 = -y/5 + 4y^2/3 + 3232y^3/15 - 14970y^4/5 + 22144y^5/3 - 70144y^6/15$	$+ \log(4y) * (-8y^2 + 96y^3 - 128y^4)$
$+ \log(4y) * (-32y^3 - 768y^4 + 2560y^5 - 20464y^6)$	$+ 1/\hat{e}_1 * (-8y^2 + 96y^3 - 128y^4)$
$+ 1/\hat{e}_0 * (-32y^3 + 768y^4 - 2560y^5 + 20464y^6)$	$\hat{S}_{15} = -2y/3 - 80y^2/3 - 128y^3 + 2816y^4/3$
$\hat{S}_2 = -y/5 + 8y^2/15 + 540y^3/15 - 11252y^4/15 + 6025y^5/3$	$+ \log(4y) * (-16y^2 + 384y^3 - 1280y^4 + 1024y^5)$
$+ \log(4y) * (-32y^3/5 - 384y^4/5 + 5128y^5/5)$	$+ 1/\hat{e}_1 * (-16y^2 + 384y^3 - 1280y^4 + 1024y^5)$
$+ 1/\hat{e}_0 * (-32y^2/5 + 384y^3/5 - 5128y^4/5)$	$\hat{S}_{16} = 12y - 64y^2 + 48y^3$
$\hat{S}_3 = -y/5$	$+ \log(4y) * (2y - 24y^2 + 32y^3)$
$\hat{S}_4 = -y/5 - 4y^2/15$	$+ 1/\hat{e}_0 * (-2y + 24y^2 - 32y^3)$
$\hat{S}_5 = -y/5 - 4y^2/15 - 16y^3/15$	$\hat{S}_{17} = 8y - 8y^2$
$\hat{S}_6 = -y/5 + 16y^3/5 - 64y^4/5$	$+ \log(4y) * (2y - 8y^2)$
$\hat{S}_7 = -y/5 + 8y^2/15 + 144y^3/5 - 192y^4 + 640y^5/3$	$+ 1/\hat{e}_0 * (-2y + 8y^2)$
$+ \log(4y) * (32y^3/5 - 384y^4/5 + 5128y^5/5)$	$\hat{S}_{18} = 108(4y) * (2y)$
$+ 1/\hat{e}_1 * (32y^3/5 - 384y^4/5 + 5128y^5/5)$	$+ 1/\hat{e}_0 * (-2y)$
$\hat{S}_8 = -2y/3 - 128y^2/3 + 1672y^3 - 10965y^4/3 + 66400y^5/3$	$\hat{S}_{19} = 108(4y) * (2y - 8y^2)$
$+ \log(4y) * (-16y^2 + 384y^3 - 1280y^4 + 1024y^5)$	$+ 1/\hat{e}_1 * (2y - 8y^2)$
$+ 1/\hat{e}_0 * (-16y^2 - 384y^3 + 1280y^4 - 1024y^5)$	$\hat{S}_{20} = 8y^2$
$\hat{S}_9 = -2y/3 - 136y^2/3 + 2272y^3 - 7064y^4/3$	$+ \log(4y) * (2y - 8y^2)$
$+ \log(4y) * (-8y^2 + 96y^3 - 128y^4)$	$+ 1/\hat{e}_1 * (2y - 8y^2)$
$+ 1/\hat{e}_0 * (8y^2 - 96y^3 + 128y^4)$	$\hat{S}_{21} = 32y^2 - 48y^3$
$\hat{S}_{10} = -2y/3 - 12y^2/3 + 16y^3$	$+ \log(4y) * (2y - 24y^2 + 32y^3)$
$+ \log(4y) * (-8y^2/3 + 32y^3/3)$	$+ 1/\hat{e}_1 * (2y - 24y^2 + 32y^3)$
$+ 1/\hat{e}_0 * (8y^2/3 - 32y^3/3)$	$\hat{S}_{22} = 72y^2 - 848y^3/3 + 704y^4/3$
$\hat{S}_{11} = -2y/3$	$+ \log(4y) * (2y - 48y^2 + 160y^3 - 128y^4)$
$\hat{S}_{12} = -2y/3 - 8y^2/3$	$+ 1/\hat{e}_1 * (2y - 48y^2 + 160y^3 - 128y^4)$

$\{[-1, 4, -1, 0]\}$	$\hat{s}_{23} = 128y^2 - 928y^3 + 5504y^4/3 - 3200y^5/3$	$[-1, -1, 4, 0]$	$\hat{s}_{58} = 7409/22880 + 563/(3021880y^4) - 1063/(18144y^3) + 2021/(6048y^2)$
	$+1/\hat{e}_1 * ( -2y - 80y^2 + 480y^3 - 896y^4 + 512y^5 )$		$-1931/(3240y)$
$\{[-1, -3, 0, 0]\}$	$\hat{s}_{24} = * ( 2y - 80y^2 + 480y^3 - 896y^4 + 512y^5 )$	$[-1, \hat{e}_1, 1]$	$+1/\hat{e}_1 * ( -1/9 - 1/(2304y^4) + 5/(288y^3) - 5/(48y^2) + 7/(36y) )$
$\{[-1, -2, 0, 0]\}$	$\hat{s}_{25} = 1/\hat{e}_0$	$[-1, -3, -3, 1]$	$\hat{s}_{64} = -y/3 + 10y^2 - 5024y^3/3 + 34880y^4/3 - 21632y^5 + 35072y^6/3$
$\{[-1, -1, 0, 0]\}$	$\hat{s}_{26} = 2 - 1/\hat{e}_1$		$+1/\hat{e}_0 * ( -320y^3 + 3200y^4 - 7680y^5 + 5120y^6 )$
$\{[-1, -3, 1, 0]\}$	$\hat{s}_{32} = 1/2 - 1/(4y)$	$[-1, -2, -3, 1]$	$+1/\hat{e}_0 * ( 320y^3 - 3200y^4 + 7680y^5 - 5120y^6 )$
$\{[-1, -2, 1, 0]\}$	$\hat{s}_{33} = 1/2 + 1/(2y)$	$[-1, -2, 1]$	$\hat{s}_{65} = -y/3 + 16y^2/3 - 416y^3 + 4352y^4/3 - 3200y^5/3$
	$+1/\hat{e}_1 * ( -1/(4y) )$		$+1/\hat{e}_0 * ( -96y^3 + 512y^4 - 512y^5 )$
$\{[-1, -1, 1, 0]\}$	$\hat{s}_{34} = 13/18 - 2/(9y)$	$[-1, -1, -3, 1]$	$+1/\hat{e}_0 * ( 96y^3 - 512y^4 + 512y^5 )$
	$+1/\hat{e}_1 * ( -1/3 + 1/(12y) )$		$+1/\hat{e}_0 * ( -16y^3 + 32y^4 )$
$\{[-1, -3, 2, 0]\}$	$\hat{s}_{40} = 1/6 + 13/(48y^2) + 1/(4y)$	$[-1, 0, -3, 1]$	$+1/\hat{e}_0 * ( 16y^3 - 32y^4 )$
	$+1/\hat{e}_1 * ( -1/(8y^2) )$		
$\{[-1, -2, 2, 0]\}$	$\hat{s}_{41} = 1/4 - 1/(6y^2) + 5/(12y)$	$[-1, 1, -3, 1]$	$\hat{s}_{67} = -y/3 - 2y^2/3$
	$+1/\hat{e}_1 * ( 1/(16y^2) - 1/(4y) )$		$\hat{s}_{68} = -y/3 - 2y^2/3$
$\{[-1, -1, 2, 0]\}$	$\hat{s}_{42} = 149/300 + 23/(600y^2) - 41/(100y)$	$[-1, 2, -3, 1]$	$\hat{s}_{69} = -y/3 - 16y^3/3$
	$+1/\hat{e}_1 * ( -1/5 - 1/(6y^2) + 3/(20y) )$		$\hat{s}_{70} = -y/3 + 2y^2 - 40y^3 + 176y^4/3$
$\{[-1, -3, 3, 0]\}$	$\hat{s}_{48} = 1/10 - 41/(320y^3) + 67/(160y^2) + 3/(16y)$		$+1/\hat{e}_0 * ( -16y^3 + 32y^4 )$
	$+1/\hat{e}_1 * ( 3/(64y^3) - 7/(16y^2) )$	$[-1, 4, -3, 1]$	$+1/\hat{e}_1 * ( -16y^3 + 16y^2/3 - 112y^3 + 512y^4/3 - 128y^5/3 )$
$\{[-1, -2, 3, 0]\}$	$\hat{s}_{49} = 1/6 + 23/(480y^3) - 77/(160y^2) + 109/(240y)$		$+1/\hat{e}_0 * ( -96y^3 + 512y^4 - 512y^5 )$
	$+1/\hat{e}_1 * ( -1/(64y^3) + 3/(16y^2) - 1/(4y) )$	$[-1, -3, -2, 1]$	$+1/\hat{e}_1 * ( -96y^3 + 512y^4 - 512y^5 )$
$\{[-1, -1, 3, 0]\}$	$\hat{s}_{50} = 383/980 - 11/(1470y^3) + 1303/(7840y^2) - 1219/(2352y)$		$+1/\hat{e}_0 * ( 36y^2 - 192y^3 + 192y^4 )$
	$+1/\hat{e}_1 * ( -1/7 + 1/(448y^3) - 3/(56y^2) + 5/(28y) )$		$+1/\hat{e}_0 * ( -36y^2 + 192y^3 - 192y^4 )$
$\{[-1, -2, 4, 0]\}$	$\hat{s}_{56} = 1/14 + 1303/(26880y^4) - 529/(1120y^3) + 673/(1680y^2) + 1/(6y)$	$[-1, -2, -2, 1]$	$\hat{s}_{73} = -2y + 48y^2 - 48y^3$
	$+1/\hat{e}_1 * ( -1/(64y^4) + 3/(16y^3) - 1/(4y^2) )$		$+1/\hat{e}_0 * ( 16y^2 - 32y^3 )$
$\{[-1, -2, 4, 0]\}$	$\hat{s}_{57} = 1/8 - 11/(840y^4) + 317/(1120y^3) - 357/(672y^2) + 139/(280y)$		$+1/\hat{e}_0 * ( -16y^2 + 32y^3 )$
	$+1/\hat{e}_1 * ( 1/(256y^4) - 3/(32y^3) + 5/(16y^2) - 1/(4y) )$		

TABLE II—Continued

$\hat{S}_{74} = -2y + 4y^2$	$\hat{S}_{86} = \frac{4y}{\log(4y)} * (-2y + 20y^2 - 48y^3 + 32y^4)$
$+ \log(4y) * (-4y^2)$	
$+1/\hat{e}_0 * (-4y^2)$	
$\hat{S}_{75} = -2y$	$\hat{S}_{87} = \frac{4y}{\log(4y)} * (-2y + 224y^2/3 + 8224y^3/25 + 38144y^4/75 + 18072y^5/15$
$\hat{S}_{76} = -2y + 4y^2$	$+ \log(4y) * (-2y + 32y^2 - 672y^3/5 + 1024y^4/5 - 512y^5/5)$
$+ \log(4y) * (-4y^2)$	$+1/\hat{e}_1 * (-2y + 32y^2 - 672y^3/5 + 1024y^4/5 - 512y^5/5)$
$+1/\hat{e}_1 * (-4y^2)$	
$\hat{S}_{77} = -2y + 16y^3$	$\hat{S}_{88} = -1/2 + 1/(2*\hat{e}_0)$
$+ \log(4y) * (-16y^2 - 32y^3)$	
$+1/\hat{e}_1 * (-16y^2 - 32y^3)$	
$\hat{S}_{78} = -2y - 12y^2 + 192y^3 - 224y^4$	$\hat{S}_{89} = 1$
$+ \log(4y) * (36y^2 - 192y^3 + 192y^4)$	$\hat{S}_{90} = 1 - 1/2*\hat{e}_1$
$+1/\hat{e}_1 * (36y^2 - 192y^3 + 192y^4)$	
$\hat{S}_{79} = -2y - 32y^2 + 2432y^3/3 - 2304y^4 + 4864y^5/3$	$\hat{S}_{91} = 1/4 + 3/(8y)$
$+ \log(4y) * (64y^2 - 640y^3 + 1536y^4 - 1024y^5)$	
$+1/\hat{e}_1 * (64y^2 - 640y^3 + 1536y^4 - 1024y^5)$	$\hat{S}_{92} = 1/3 + 5/(12y)$
$\hat{S}_{80} = -8y + 4y^2$	$\hat{S}_{93} = 1/2 + 1/(4y)$
$+ \log(4y) * (-2y + 4y^2)$	
$+1/\hat{e}_0 * (2y - 4y^2)$	$\hat{S}_{94} = 1/12 - 1/(3y)$
$\hat{S}_{81} = \log(4y) * (-2y)$	$\hat{S}_{95} = 1/12 + 1/(3y)$
$+1/\hat{e}_0 * (2y)$	
$\hat{S}_{82} = y * z$	$\hat{S}_{96} = 1/16 + 2853/(17920y^4) - 1587/(240y^3) + 1041/(2240y^2) + 9/(56y)$
$\hat{S}_{83} = 4y$	$+1/\hat{e}_1 * (-27/(512y^5) + 9/(32y^3) - 9/(32y^2))$
$+ \log(4y) * (-2y)$	$\hat{S}_{97} = 1/9 - 4189/(80640y^6) + 479/(1008y^3) - 853/(840y^2) + 1301/(2520y)$
$+1/\hat{e}_1 * (-2y)$	
$\hat{S}_{84} = 4y - 8y_2$	$\hat{S}_{98} = 1/(64y^4) - 5/(32y^3) + 3/(8y^2) - 1/(4y)$
$+ \log(4y) * (-2y + 4y^2)$	
$+1/\hat{e}_1 * (-2y + 4y^2)$	$\hat{S}_{99} = 1/16 + 2853/(25200 + 563/(80640y^6) - 2147/(20160y^3)$
$\hat{S}_{85} = 4y - 20y^2/9 + 208y^3/9$	$+ 2057/(4800y^2) - 7913/(1260y)$
$+ \log(4y) * (-2y + 32y^2/3 - 32y^3/3)$	
$+1/\hat{e}_1 * (-2y + 32y^2/3 - 32y^3/3)$	$+1/\hat{e}_1 * (-1/10 - 1/(512y^6) + 1/(32y^3) - 21/(160y^2) + 1/(5y))$

TABLE III

## TABLES ON AXIAL GAUGES

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Table III(a) Axial gauge integrals with $K=0$		Table III(d) Axial gauge integrals with $K=3$	
$[0,-1,-1,0]$	$\hat{S}_{18} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (-2y)$	$[1,-3,0,0]$	$\hat{S}_{24} = 6/\hat{e}_0 - 6/\hat{e}_1$
$[0,-2,0,0]$	$\hat{S}_{25} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[1,-2,0,0]$	$\hat{S}_{25} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[0,-1,-1,0]$	$\hat{S}_{32} = 1/\hat{e}_0 * (1/(4y)) + 1/\hat{e}_1 * (-1/(4y))$	$[1,-3,-1,0]$	$\hat{S}_{32} = 1/\hat{e}_0 * (1/(4y)) + 1/\hat{e}_1 * (-1/(4y))$
$[1,-2,-1,0]$	$\hat{S}_{17} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$	$[1,-3,-1,1]$	$\hat{S}_{66} = 1/\hat{e}_0 * (-16y^3 + 32y^4) + 1/\hat{e}_1 * (16y^3 - 32y^4)$
$[1,-1,-1,0]$	$\hat{S}_{18} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$	$[1,-3,-2,1]$	$\hat{S}_{72} = 1/\hat{e}_0 * (12y^2) + 1/\hat{e}_1 * (-12y^2)$
$[1,-1,0,0]$	$\hat{S}_{24} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[1,-2,-2,1]$	$\hat{S}_{73} = 1/\hat{e}_0 * (48y^2 - 32y^3) + 1/\hat{e}_1 * (-48y^2 + 32y^3)$
$[1,-2,0,0]$	$\hat{S}_{25} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[1,-1,-2,1]$	$\hat{S}_{74} = 1/\hat{e}_0 * (12y^2) + 1/\hat{e}_1 * (-12y^2)$
$[1,-3,1,0]$	$\hat{S}_{32} = 1/\hat{e}_0 * (1/(4y)) + 1/\hat{e}_1 * (-1/(4y))$	$[1,-3,-1,1]$	$\hat{S}_{80} = 1/\hat{e}_0 * (-18y + 4y^2) + 1/\hat{e}_1 * (18y - 4y^2)$
$[1,-1,-2,1]$	$\hat{S}_{74} = 1/\hat{e}_0 * (sy^2) + 1/\hat{e}_1 * (-sy^2)$	$[1,-2,-1,1]$	$\hat{S}_{81} = 1/\hat{e}_0 * (-6y) + 1/\hat{e}_1 * (6y)$
$[1,-2,-1,1]$	$\hat{S}_{84} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$	$[1,-3,0,1]$	$\hat{S}_{88} = -3/2 * 1/\hat{e}_0 + 3/2 * 1/\hat{e}_1$
$[1,-1,0,1]$	$\hat{S}_{88} = -1/2 * 1/\hat{e}_0 + 1/2 * 1/\hat{e}_1$	Table III(e) Axial gauge integrals with $K=4$	
Table III(c) Axial gauge integrals with $K=2$		$[1,-1,-3,0]$	$\hat{S}_2 = 1/\hat{e}_0 * (-35y^3/5 + 38y^4/5 - 512y^5/5)$
$[12,-1,-2,0]$	$\hat{S}_{10} = 1/\hat{e}_0 * (8y^2/3 - 32y^3/3) + 1/\hat{e}_1 * (-8y^2/3 + 32y^3/3)$	$[1,-3,-2,0]$	$\hat{S}_8 = 1/\hat{e}_0 * (16y^2 - 64y^3) + 1/\hat{e}_1 * (-16y^2 + 64y^3)$
$[12,-1,-1,0]$	$\hat{S}_{16} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$	$[1,-2,-2,0]$	$\hat{S}_9 = 1/\hat{e}_0 * (48y^2 - 256y^3 + 128y^4) + 1/\hat{e}_1 * (-48y^2 + 256y^3 - 128y^4)$
$[12,-2,-1,0]$	$\hat{S}_{17} = 1/\hat{e}_0 * (-8y + 8y^2) + 1/\hat{e}_1 * (8y - 8y^2)$	$[1,-1,-2,0]$	$\hat{S}_{10} = 1/\hat{e}_0 * (16y^2 - 64y^3) + 1/\hat{e}_1 * (-16y^2 + 64y^3)$
$[12,-1,-1,0]$	$\hat{S}_{18} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$	$[1,-3,-1,0]$	$\hat{S}_{16} = 1/\hat{e}_0 * (-72y + 148y^2 - 32y^3) + 1/\hat{e}_1 * (72y - 148y^2 + 32y^3)$
$[12,-3,0,0]$	$\hat{S}_{24} = 3/\hat{e}_0 - 3/\hat{e}_1$	$[1,-2,-1,0]$	$\hat{S}_{17} = 1/\hat{e}_0 * (-32y + 48y^2) + 1/\hat{e}_1 * (32y - 48y^2)$
$[12,-2,0,0]$	$\hat{S}_{25} = 1/\hat{e}_0 - 1/\hat{e}_1$	$[1,-1,-1,0]$	$\hat{S}_{18} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$
$[12,-3,1,0]$	$\hat{S}_{32} = 1/\hat{e}_0 * (1/(4y)) + 1/\hat{e}_1 * (-1/(4y))$	$[1,-3,0,0]$	$\hat{S}_{24} = 10/\hat{e}_0 - 10/\hat{e}_1$
$[12,-2,-2,1]$	$\hat{S}_{73} = 1/\hat{e}_0 * (sy^2) + 1/\hat{e}_1 * (-sy^2)$	$[1,-2,-0,0]$	$\hat{S}_{25} = 1/\hat{e}_0 - 1/\hat{e}_1$
$[12,-1,-2,1]$	$\hat{S}_{74} = 1/\hat{e}_0 * (-4y) + 1/\hat{e}_1 * (4y)$	$[1,-3,1,0]$	$\hat{S}_{32} = 1/\hat{e}_0 * (1/(4y)) + 1/\hat{e}_1 * (-1/(4y))$
$[12,-3,-1,1]$	$\hat{S}_{80} = 1/\hat{e}_0 * (8y^2 - 32y^3) + 1/\hat{e}_1 * (-8y^2 + 32y^3)$	$[1,-2,-2,1]$	$\hat{S}_{73} = 1/\hat{e}_0 * (144y^2 - 128y^3) + 1/\hat{e}_1 * (-144y^2 + 128y^3)$
$[12,-2,-1,1]$	$\hat{S}_{81} = 1/\hat{e}_0 * (-4y) + 1/\hat{e}_1 * (4y)$	$[1,-1,-2,1]$	$\hat{S}_{74} = 1/\hat{e}_0 * (16y^2) + 1/\hat{e}_1 * (-16y^2)$
$[12,-1,0,1]$	$\hat{S}_{88} = -1/\hat{e}_0 + 1/\hat{e}_1$	$[1,-3,-1,1]$	$\hat{S}_{80} = 1/\hat{e}_0 * (-48y + 16y^2) + 1/\hat{e}_1 * (48y - 16y^2)$
Table III(d) Axial gauge integrals with $K=3$		$[1,-2,-1,1]$	$\hat{S}_{81} = 1/\hat{e}_0 * (-8y) + 1/\hat{e}_1 * (8y)$
$[13,-2,-2,0]$	$\hat{S}_9 = 1/\hat{e}_0 * (8y^2 - 32y^3) + 1/\hat{e}_1 * (-8y^2 + 32y^3)$	$[1,-3,0,1]$	$\hat{S}_{88} = -2/\hat{e}_0 + 2/\hat{e}_1$
$[13,-1,-2,0]$	$\hat{S}_{10} = 1/\hat{e}_0 * (8y^2 - 32y^3) + 1/\hat{e}_1 * (-8y^2 + 32y^3)$		
$[13,-3,-1,0]$	$\hat{S}_{16} = 1/\hat{e}_0 * (-18y + 24y^2) + 1/\hat{e}_1 * (18y - 24y^2)$		
$[13,-2,-1,0]$	$\hat{S}_{17} = 1/\hat{e}_0 * (-18y + 24y^2) + 1/\hat{e}_1 * (18y - 24y^2)$		
$[13,-1,-1,0]$	$\hat{S}_{18} = 1/\hat{e}_0 * (-2y) + 1/\hat{e}_1 * (2y)$		

TABLE IV  
Light-Cone Gauge Integrals (P.V.) with  $K = -2$

$[-2, -3, -3, 0]$	$\hat{L}_0 = -573/70 + 2/\hat{e}$		
$[-2, -2, -3, 0]$	$\hat{L}_1 = -69/10 + 2/\hat{e}$	$[-2, -1, -3, 1]$	$\hat{L}_{66} = -25/6 + 2/\hat{e}$
$[-2, -1, -3, 0]$	$\hat{L}_2 = -137/30 + 2/\hat{e}$	$[-2, -3, -2, 1]$	$\hat{L}_{72} = -43/6 + 2/\hat{e}$
$[-2, -3, -2, 0]$	$\hat{L}_8 = -227/30 + 2/\hat{e}$	$[-2, -2, -2, 1]$	$\hat{L}_{73} = -17/3 + 2/\hat{e}$
$[-2, -2, -2, 0]$	$\hat{L}_9 = -37/6 + 2/\hat{e}$	$[-2, -1, -2, 1]$	$\hat{L}_{74} = -3 + 2/\hat{e}$
$[-2, -1, -2, 0]$	$\hat{L}_{10} = -11/3 + 2/\hat{e}$	$[-2, -3, -1, 1]$	$\hat{L}_{80} = -6 + 2/\hat{e}$
$[-2, -3, -1, 0]$	$\hat{L}_{16} = -20/3 + 2/\hat{e}$	$[-2, -2, -1, 1]$	$\hat{L}_{81} = -4 + 2/\hat{e}$
$[-2, -2, -1, 0]$	$\hat{L}_{17} = -5 + 2/\hat{e}$	$[-2, -1, -1, 1]$	$\hat{L}_{82} = 2/\hat{e}$
$[-2, -1, -1, 0]$	$\hat{L}_{18} = -2 + 2/\hat{e}$	$[-2, -3, 0, 1]$	$\hat{L}_{88} = -3 + 2/\hat{e}$
$[-2, -3, 0, 0]$	$\hat{L}_{24} = -5 + 2/\hat{e}$	$[-2, -2, 0, 1]$	$\hat{L}_{89} = -1 + 1/\hat{e}$
$[-2, -2, 0, 0]$	$\hat{L}_{25} = -2 + 2/\hat{e}$	$[-2, -1, 0, 1]$	$\hat{L}_{90} = -1 + 1/\hat{e}$
$[-2, -1, 0, 0]$	$\hat{L}_{26} = 1/\hat{e}$	$[-2, -3, 1, 1]$	$\hat{L}_{96} = -5/2 + 1/\hat{e}$
$[-2, -3, 1, 0]$	$\hat{L}_{32} = -3/2 + 1/\hat{e}$	$[-2, -2, 1, 1]$	$\hat{L}_{97} = -5/2 + 1/\hat{e}$
$[-2, -2, 1, 0]$	$\hat{L}_{33} = -2 + 1/\hat{e}$	$[-2, -1, 1, 1]$	$\hat{L}_{98} = -11/6 + 1/\hat{e}$
$[-2, -1, 1, 0]$	$\hat{L}_{34} = -3/2 + 1/\hat{e}$	$[-2, -3, 2, 1]$	$\hat{L}_{104} = -10/3 + 1/\hat{e}$
$[-2, -3, 2, 0]$	$\hat{L}_{40} = -3 + 1/\hat{e}$	$[-2, -2, 2, 1]$	$\hat{L}_{105} = -37/12 + 1/\hat{e}$
$[-2, -2, 2, 0]$	$\hat{L}_{41} = -17/6 + 1/\hat{e}$	$[-2, -1, 2, 1]$	$\hat{L}_{106} = -137/60 + 1/\hat{e}$
$[-2, -1, 2, 0]$	$\hat{L}_{42} = -25/12 + 1/\hat{e}$	$[-2, -3, 3, 1]$	$\hat{L}_{112} = -227/60 + 1/\hat{e}$
$[-2, -3, 3, 0]$	$\hat{L}_{48} = -43/12 + 1/\hat{e}$	$[-2, -2, 3, 1]$	$\hat{L}_{113} = -69/20 + 1/\hat{e}$
$[-2, -2, 3, 0]$	$\hat{L}_{49} = -197/60 + 1/\hat{e}$	$[-2, -1, 3, 1]$	$\hat{L}_{114} = -363/140 + 1/\hat{e}$
$[-2, -1, 3, 0]$	$\hat{L}_{50} = -49/20 + 1/\hat{e}$	$[-2, -3, 4, 1]$	$\hat{L}_{120} = -573/140 + 1/\hat{e}$
$[-2, -3, 4, 0]$	$\hat{L}_{56} = -79/20 + 1/\hat{e}$	$[-2, -2, 4, 1]$	$\hat{L}_{121} = -1041/280 + 1/\hat{e}$
$[-2, -2, 4, 0]$	$\hat{L}_{57} = -503/140 + 1/\hat{e}$	$[-2, -1, 4, 1]$	$\hat{L}_{122} = -7129/2520 + 1/\hat{e}$
$[-2, -1, 4, 0]$	$\hat{L}_{58} = -761/280 + 1/\hat{e}$		
$[-2, -3, -3, 1]$	$\hat{L}_{64} = -79/10 + 2/\hat{e}$		
$[-2, -2, -3, 1]$	$\hat{L}_{65} = -197/30 + 2/\hat{e}$		

TABLE V  
Light-Cone Gauge Integrals (P.V.) with  $K = -1$

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$[-1, -3, -3, 0]$	$\hat{L}_0 = -1/7$		
$[-1, -2, -3, 0]$	$\hat{L}_1 = -1/3$	$[-1, -1, -3, 1]$	$\hat{L}_{66} = -19/24 + 1/2 * 1/\hat{e}$
$[-1, -1, -3, 0]$	$\hat{L}_2 = -113/150 + 2/5 * 1/\hat{e}$	$[-1, -3, -2, 1]$	$\hat{L}_{72} = -1/4$
$[-1, -3, -2, 0]$	$\hat{L}_3 = -1/5$	$[-1, -2, -2, 1]$	$\hat{L}_{73} = -2/3$
$[-1, -2, -2, 0]$	$\hat{L}_4 = -1/2$	$[-1, -1, -2, 1]$	$\hat{L}_{74} = -1/2 + 1/\hat{e}$
$[-1, -1, -2, 0]$	$\hat{L}_{10} = -7/9 + 2/3 * 1/\hat{e}$	$[-1, -3, -1, 1]$	$\hat{L}_{80} = -1/2$
$[-1, -3, -1, 0]$	$\hat{L}_{16} = -1/3$	$[-1, -2, -1, 1]$	$\hat{L}_{81} = -2$
$[-1, -2, -1, 0]$	$\hat{L}_{17} = -1$	$[-1, -1, -1, 1]$	$\hat{L}_{82} = -1/\hat{e}^2 + 1/\hat{e} * (\log(p^2) + \gamma)$ $+ \pi^2/12 - 1/2 * (\log(p^2) + \gamma)^2$
$[-1, -1, -1, 0]$	$\hat{L}_{18} = 2 + z * 1/\hat{e}$		
$[-1, -3, 0, 0]$	$\hat{L}_{24} = -1$	$[-1, -3, 0, 1]$	$\hat{L}_{88} = -1/2 + 1/2 * 1/\hat{e}$
$[-1, -2, 0, 0]$	$\hat{L}_{25} = 1/\hat{e}$	$[-1, -2, 0, 1]$	$\hat{L}_{89} = 1$
$[-1, -1, 0, 0]$	$\hat{L}_{26} = 2 - 1/\hat{e}$	$[-1, -1, 0, 1]$	$\hat{L}_{90} = 1 - 1/2 * 1/\hat{e}$
$[-1, -3, 1, 0]$	$\hat{L}_{32} = 1/2$	$[-1, -3, 1, 1]$	$\hat{L}_{96} = 1/4$
$[-1, -2, 1, 0]$	$\hat{L}_{33} = 1/2$	$[-1, -2, 1, 1]$	$\hat{L}_{97} = 1/3$
$[-1, -1, 1, 0]$	$\hat{L}_{34} = 13/18 - 1/3 * 1/\hat{e}$	$[-1, -1, 1, 1]$	$\hat{L}_{98} = 7/12 - 1/4 * 1/\hat{e}$
$[-1, -3, 2, 0]$	$\hat{L}_{40} = 1/6$	$[-1, -3, 2, 1]$	$\hat{L}_{104} = 1/8$
$[-1, -2, 2, 0]$	$\hat{L}_{41} = 1/4$	$[-1, -2, 2, 1]$	$\hat{L}_{105} = 1/5$
$[-1, -1, 2, 0]$	$\hat{L}_{42} = 149/300 - 1/5 * 1/\hat{e}$	$[-1, -1, 2, 1]$	$\hat{L}_{106} = 157/360 - 1/6 * 1/\hat{e}$
$[-1, -3, 3, 0]$	$\hat{L}_{48} = 1/10$	$[-1, -3, 3, 1]$	$\hat{L}_{112} = 1/12$
$[-1, -2, 3, 0]$	$\hat{L}_{49} = 1/6$	$[-1, -2, 3, 1]$	$\hat{L}_{113} = 1/7$
$[-1, -1, 3, 0]$	$\hat{L}_{50} = 383/980 - 1/7 * 1/\hat{e}$	$[-1, -1, 3, 1]$	$\hat{L}_{114} = 199/560 - 1/8 * 1/\hat{e}$
$[-1, -3, 4, 0]$	$\hat{L}_{56} = 1/14$	$[-1, -3, 4, 1]$	$\hat{L}_{120} = 1/16$
$[-1, -2, 4, 0]$	$\hat{L}_{57} = 1/8$	$[-1, -2, 4, 1]$	$\hat{L}_{121} = 1/9$
$[-1, -1, 4, 0]$	$\hat{L}_{58} = 7409/22680 - 1/9 * 1/\hat{e}$	$[-1, -1, 4, 1]$	$\hat{L}_{122} = 7633/25200 - 1/10 * 1/\hat{e}$
$[-1, -3, -3, 1]$	$\hat{L}_{64} = -1/6$		
$[-1, -2, -3, 1]$	$\hat{L}_{65} = -2/5$		

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RECEIVED February 17, 1984; REVISED May 31, 1984

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